Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-10-04

Conceptual questions

Question 1. Which of the following regions in $\mathbb{R}^2$ are closed? Which of them are bounded?

(a) The entirety of $\mathbb{R}^2$
(b) The line segment connecting $(2,3)$ and $(5,-10)$, including the endpoints
(c) $x^4 + y^6 = 2$
(d) $x \geq 3$
(e) $x^2 < y \leq 4 - x^2$
(f) $x^3 + y^2 = 10$

Question 2. Explain why the system of equations

$$ye^{xy} = 4x^3\lambda$$
$$xe^{xy} = 6y^5\lambda$$
$$x^4 + y^6 = 2$$

must have *at least two* solutions.

Question 3. What happens if you try to use Lagrange multipliers to find the extrema of $f(x,y) = x$ with the constraint $y^2 = x^3$? Draw a picture. See also exercise §14.8.25.

Computations

Problem 1. The plane $4x - 3y + 8z = 5$ intersects the cone $z^2 = x^2 + y^2$ in an ellipse. Find the highest and lowest points on this ellipse (i.e. the points with extremal z values). Try doing this problem in multiple ways.